



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 36

Introduction to Options

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VALUING A FORWARD AFTER INCEPTION

- Consider a long position in a forward contract A set up at $t=0$ with maturity T and price $F(0,T)$.
- Let, at a later time t^* , a forward contract B on the same underlying for maturity at T have a forward price of $F(t^*,T)$.
- Then, both contracts A & B entail delivery of the same asset at same time ($t=T$) but at different prices $F(0,T)$ and $F(t^*,T)$.

- Clearly, if $F(0,T) < F(t^*,T)$, a long position in the original forward A will command a positive value.
- Further, this value will be the present value the difference $F(t^*,T) - F(0,T)$ so that
- $V_A(t^*,T) = [F(t^*,T) - F(0,T)]e^{-r(T-t^*)} = S_{t^*} - F(0,T)e^{-r(T-t^*)} = (S_{t^*} - S_0 e^{rt^*})$.



$t_0 = \text{JAN 1}, F_0 = 75$
LONG

$t^* = \text{APR 1}, F_{t^*} = 90$
SHORT

$T = \text{DEC 31},$
 $CF = 90 - 75 = 15$

$$V_{\text{portfolio}} = V_A - V_B = V_A = \text{PV of 15 at } t=t^* = \text{PV}(F_{t^*,T} - F_{0,T})$$

The value is positive and riskfree since 15 shall be received with certainty at $t=T$.

EXAMPLE 4: FORWARD PRICING

- On 1st January 2018, the spot price of wheat was Rs 1,200 per quintal. X took a long position in a forward contract with maturity of 9 months. The risk-free rate of interest was 24% p.a. with continuous compounding. However, on April 1, 2018, the spot price of wheat of the same quality had increased to Rs 1,400 per quintal while the riskfree rate remained unchanged. Calculate the value of the original (January 1) contract as on April 1 for X.



SOLUTION		
WHEAT PRICE		
Jan-01		1200
Apr-01		1400
RISKFREE RATE		0.24
FUTURE VALUE OF JAN PRICE		
ON APRIL 1		1274.2039
HENCE VALUE OF FORWARD		
AS ON APRIL 1		125.79614

EXAMPLE 5: FORWARD PRICING

- On 1st April 2018, a stock was expected to pay a dividend of 2.10 per share in two months ($t=2$) and in five months ($t=5$). The stock price at this date ($t=0$) was 50, and the risk-free rate of interest was 24% p.a. with continuous compounding. An investor had taken a short position in a six-month forward contract on the stock on that date. Three months later (1st July 2018), the price of the stock was 44 and the risk-free rate was still 24% per annum. Calculate the value of the original contract (April 1) as on July 1.

SOLUTION					
TIME		0.0000	2.0000	5.0000	6.0000
STOCK PRICE		50.0000			
INTEREST RATE(24%)		0.2400	0.0400	0.1000	0.1200
DISCOUNT FACTOR			-0.0400	-0.1000	
			0.9608	0.9048	
DIVIDEND			2.1000	2.1000	
PV OF DIVIDEND			2.0177	1.9002	1.1275
		3.9178			
NET STOCK PRICE AT T=0		46.0822			
FORWARD PRICE AT T=6		51.9575			

USING THE SAME METHODOLOGY WE CALCULATE THE FORWARD PRICE OF THE NEW CONTRACT

TIME		3.0000	5.0000	6.0000	
STOCK PRICE		44.0000			
INTEREST RATE(24%)		0.2400	0.0400	0.0600	
DISCOUNT FACTOR			-0.0400	-0.0600	
			0.9608	0.9418	
DIVIDEND			2.1000	0.0000	
PV OF DIVIDEND			2.0177	0.0000	1.0618
		2.0177			
NET STOCK PRICE AT T=0		41.9823			
FORWARD PRICE AT T=6		44.5784			
DIFFERENCE IN FORWARD PRICES		7.3791			
PV OF THIS DIFFERENCE		6.9494			

OPTIONS: INTRODUCTION & SALIENT FEATURES

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OPTION CONTRACT

- An option is a contract whereby the holder of the contract acquires a **RIGHT** to buy/sell a certain asset by/on a certain date for a certain price.
- Accordingly, the writer of the option has the **OBLIGATION** to sell/buy the asset by/on the said date for the said price, if the holder decides to exercise the option. He has no **DISCRETION**.

TERMINOLOGY

- **European vs American Option**
- **Call & Put Options**
- **Underlying Asset**
- **Strike Price/Exercise Price**
- **Strike Date/Maturity Date/Exercise Date**
- **Holder of the Option vs Writer of the Option**
- **Long Position vs Short Position**



MONEYNESS OF OPTIONS

• Condition	Call	Put
• $S_t > K$	In the money	Out of money
• $S_t < K$	Out of money	In the money
• $S_t = K$	At the money	At the money

VALUE OF OPTION

- Intrinsic value of an option is the amount by which it is in the money.
- For a call option $IV_{\text{call}}(t) = \max(0, S_t - K)$
- For a put option $IV_{\text{put}}(t) = \max(0, K - S_t)$
- The intrinsic value of an out of the money option is zero.
- Time Value = Option Premium – IV
- Source of Time Value of options????

NOTATION

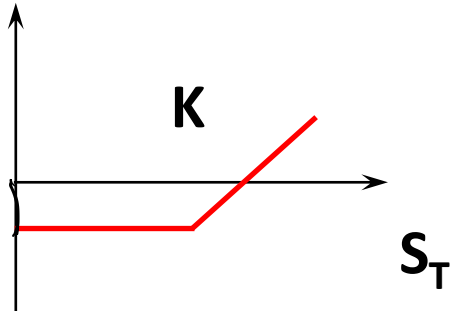
- c :European call option price
- p :European put option price
- S_0 :Stock price today
- K :Strike price
- T :Life of option
- σ :Volatility of stock price
- C :American call option price
- P :American put option price
- S_T :Stock price at option maturity
- D_0 :PV of dividends paid during life of option
- r : Risk-free rate for maturity T with cont. comp.



PROFIT DIAGRAM OF LONG CALL

- We ignore time value of money on option premium for simplicity of exposition:
- Profit Diagram:

$\pi(S_T)$



$$\pi_{\text{long call}} = \begin{cases} -c & \text{if } S_T \leq K \\ (S_T - K) - c & \text{if } S_T \geq K \end{cases}$$

$$\pi_{\text{long call}}^{\max} = \infty$$

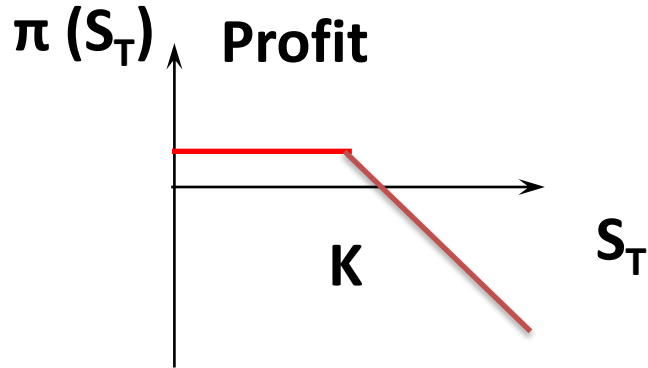
$$\pi_{\text{long call}}^{\min} = -c$$

$$S_T^{\text{bep}} = K + c$$

- Investor Perception???

PROFIT DIAGRAM OF SHORT CALL

- Profit Diagram:
- Mirror Image of Long Call:



$$\pi_{\text{short call}} = \begin{cases} +c & \text{if } S_T \leq K \\ (K - S_T) + c & \text{if } S_T \geq K \end{cases}$$

$$\pi_{\text{short call}}^{\max} = +c$$

$$\pi_{\text{short call}}^{\min} = -\infty$$

$$S_T^{\text{bep}} = K + c$$

- Investor Perception???

RELATION BETWEEN PREMIUM & STRIKE PRICE FOR CALLS & PUTS

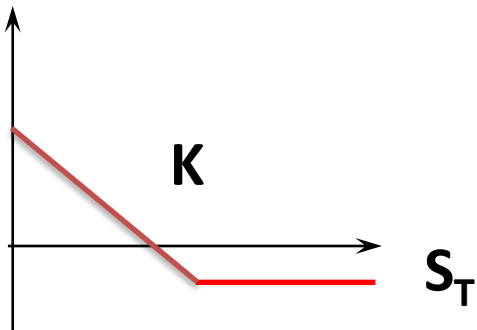
- A call gives the right to the holder to buy the asset at the exercise price.
- Clearly, lower the exercise price, the more valuable is the call and hence, higher its premium.
- Converse is the case with puts, where the premium is directly related to strike price.



PROFIT DIAGRAM OF LONG PUT

- Profit Diagram:

$\pi(S_T)$ Profit



$$\pi_{\text{long put}} = \begin{cases} (K - S_T) - p & \text{if } S_T \leq K \\ -p & \text{if } S_T \geq K \end{cases}$$

$$\pi_{\text{long put}}^{\max} = K - p$$

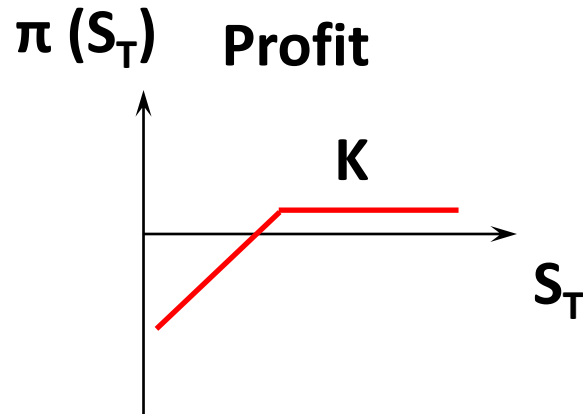
$$\pi_{\text{long put}}^{\min} = -p$$

$$S_T^{\text{bep}} = K - p$$

- Investor Perception???

PROFIT DIAGRAM OF SHORT PUT

- Profit Diagram:



$$\pi_{\text{short put}} = \begin{cases} -(K - S_T) + p & \text{if } S_T \leq K \\ +p & \text{if } S_T \geq K \end{cases}$$

$$\pi_{\text{short put}}^{\max} = p$$

$$\pi_{\text{short put}}^{\min} = -(K - p)$$

$$S_T^{\text{bep}} = K - p$$

PUT CALL PARITY

	$t=0$	$t=T$	
PORTFOLIO A		$S_T < K$	$S_T > K$
BUY CALL	$-c$	0	$S_T - K$
INVEST	$-Ke^{(-rT)}$	K	K
TOTAL	$-c - Ke^{(-rT)}$	K	S_T



	$t=0$	$t=T$	
PORTFOLIO A		$S_T < K$	$S_T > K$
TOTAL	$-c - Ke^{(-rT)}$	K	S_T
PORTFOLIO B			
BUY STOCK	$-S_0$	S_T	S_T
BUY PUT	$-p$	$(K - S_T)$	0
TOTAL	$-S_0 - p$	K	S_T
$c + Ke^{(-rT)} = S_0 + p$			